

# TURBULENCE INTENSITY IN DILUTE TWO-PHASE FLOWS—2

## TEMPERATURE FLUCTUATIONS IN PARTICLE-LADEN DILUTE FLOWS

## L. P. YARIN and G. HETSRONI

Department of Mechanical Engineering, Technion-Israel Institute of Technology, Haifa, Israel

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Abstract—We deal with the temperature fluctuations of the carrier fluid and particles in turbulent two-phase flows. The analysis of the temperature fluctuations is based on the modified mixing-length theory, which was used previously to evaluate the intensity of turbulent fluctuations in two-phase polydisperse flows. The dependences of the temperature fluctuations of the interacting phases on the particle sizes, the mass content of the admixture and its composition and physical properties have been predicted. The effect of heat transfer by radiation on the intensity of temperature fluctuations in two-phase flow is estimated.

Key Words: two-phase flow, turbulence, particles, temperature fluctuations

## **1. INTRODUCTION**

In the preceding paper (Yarin & Hetsroni 1994, this issue, pp. 1–15), hereafter referred to as part 1), we examined the effect of the particle size distribution on the turbulence of the carrier fluid. Here we expand the problem to include the heat transfer and temperature distributions. In particular, we examine here the temperature fluctuations around a temporal or space average. The latter, besides being of scientific interest, is of intense practical importance since rates of reactions are nonlinearly dependent on the *local* temperature. For example, the rate of NO<sub>x</sub> formation is dependent on the *local* temperature, depending on the temperature fluctuations around the average (Vulis *et al.* 1968).

There are only a few investigations on temperature fluctuations in two-phase flows. Soo (1990) formulated the problem and considered some statistical properties of the temperature fluctuations. Shraiber *et al.* (1990) also evaluated the level of temperature fluctuations and heat transfer. In particular, they showed that an increase in particle size and heat capacity leads to a decrease in energy transfer from the fluid to the particles. Lisin (1988) gives some data on the radiation heat transfer effect on the intensity of temperature fluctuations. It was shown that radiation leads to a decrease in turbulent energy transfer.

Here we study the temperature fluctuations in a two-phase flow consisting of a carrier fluid and polydisperse or monodisperse particles. Convective heat transfer and radiation will be considered. The effects of various physical properties of the particles and the carrier fluid on the temperature fluctuations will be considered, as well as the effect of particle sizes.

## 2. ANALYSIS

As in part 1, consider the steady flow of incompressible nonabsorbing fluid, laden with spherical particles with the diameters,  $d_1, d_2, \ldots, d_n$ , at number concentrations  $k_1, k_2, \ldots, k_n$ , respectively. The fluid element of the carrier gas keeps its identity during a certain period, at which it moves one mixing length. As in part 1, we use a modified Prandtl's mixing-length theory. If the particles are relatively small, and their time constant is smaller than a hydrodynamic characteristic time, one can assume that the average velocities of the particles are approximately equal to that of the fluid.

However, due to radiation, the average temperature of the particles is not equal to that of the fluid. Let  $T_p$  be the temperature of the particles and  $T_w$  be the temperature of the walls surrounding the flow. Then, when  $T_w > T_p$ , the temperature of the particles exceeds that of the carrier fluid. At a high enough intensity of convective heat transfer (which is characteristic of fine particles) and a moderate intensity of radiation, the average temperature of the particles may be considered to be equal to that of the carrier fluid.

Further, we restrict the analysis to flows with slightly varying longitudinal temperature, where the relative *average* temperature of the carrier fluid and the particles is negligibly small (developed turbulent flow in jets etc.).

The equations describing the convervation of momentum of the fluid element and the particles included in it were given in part 1.

In addition, we have the energy balances as follows:

$$\frac{c}{\gamma} \left[ \sum_{i=1}^{n} m_{p}^{(i)} k_{i} \right] \frac{dT'}{dt} = -4 \sum_{i=1}^{n} \alpha_{i} f_{p}^{(i)} k_{i} [T' - T_{p}^{\prime(i)}]$$
[1]

and

$$m_{\rm p}^{(i)} c_{\rm p}^{(i)} \frac{{\rm d}T_{\rm p}^{\prime(i)}}{{\rm d}t} = 4\alpha_i f_{\rm p}^{(i)} (T' - T_{\rm p}^{\prime(i)}) - 4q_{\rm R} f_{\rm p}^{(i)}, \qquad [2]$$

where T' is the fluctuation of the temperature of the fluid and  $T'_p$  is that of the particles, t is the time,  $m_p^{(i)}$  is the mass of a particle in size group i,  $c_p^{(i)}$  is its specific heat, c is the specific heat of the carrier fluid,  $\alpha$  is the heat transfer coefficient, f is the cross section of the particles,  $\gamma_i$  is the mass content of particles of size  $d_i$  and  $\gamma$  is the total mass content of the particles, and  $q_R$  is the radiative heat exchange between the particles and the walls.

The initial conditions for the velocities will be taken as in part 1, with the temperature initial conditions

$$t = 0, \quad T' = T'_0, \quad T'_p = T'_{p0} = 0.$$
 [3]

Note, that these conditions actually describe the physical mechanism of the fluctuational motion in turbulent two-phase flows. First, the carrier fluid fluctuation begins and afterwards momentum and heat are transferred to the particles due to viscous and thermal interaction. Thus, the particles' temperature fluctuations arise after temperature fluctuations of the fluid. Therefore, initial particle temperature fluctuations are equal to zero since the characteristic time of thermal interaction is nonzero. It is emphasized that the initial conditions used in part 1 and above lead to physically reasonable results, as well as to fairly good agreement with experimental data (Abramovich *et al.* 1984).

In a particular case, when  $q_{\rm R} = 0$  in [2], one can multiply it by  $k_i$  and perform a summation:

$$\sum_{i=1}^{n} m_{\rm p}^{(i)} c_{\rm p}^{(i)} k_i \frac{\mathrm{d}T_{\rm p}^{\prime(i)}}{\mathrm{d}t} = 4 \sum_{i=1}^{n} \alpha_i f_{\rm p}^{(i)} k_i [T' - T_{\rm p}^{\prime(i)}].$$
<sup>[4]</sup>

Adding this to [1], one arrives at the energy balance equation in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[T'c\gamma^{-1}\sum_{i=1}^{n}m_{\mathrm{p}}^{(i)}k_{i}+\sum_{i=1}^{n}m_{\mathrm{p}}^{(i)}c_{\mathrm{p}}^{(i)}k_{i}T_{\mathrm{p}}^{\prime(i)}\right]=0.$$
[5]

Integrating [5] with the initial conditions [3] we obtain

$$\bar{T}' + \sum_{i=1}^{n} \gamma_i \bar{c}^{(i)} \bar{T}'_{p}^{(i)} = 1,$$
[6]

where  $\overline{T} = T'/T'_0$  and  $\overline{T'_p}^{(i)} = T'_p^{(i)}/T'_0$  are the nondimensional temperatures and  $\overline{c}^{(i)} = c_p^{(i)}/c$  is a nondimensional specific heat.

The velocity fluctuations are given in part 1. For the simple case when  $T = T_p^{(i)}$  and  $\bar{c}^{(i)} = \bar{c}$ , the temperature fluctuation is

$$\bar{T}' = (1 + \gamma \bar{c})^{-1},$$
 [7]

which is analogous to the velocity fluctuations in part 1, meaning that the temperature fluctuations of the carrier fluid depend on the total mass content of the admixture  $\gamma$  and the ratio of specific

heats. The latter implies that when the particles' specific heat is smaller than that of the fluid, i.e. when  $\bar{c} < 1$ , the temperature fluctuation is *larger* than the velocity fluctuations, as shown in figure 1. For most metals in air we have  $\bar{c} < 1$ , and a relatively small heat flux to the particles causes a strong increase in their temperature.

## 3. MONODISPERSE MIXTURE

Consider the temperature fluctuations in a two-phase flow, where the particles are of uniform size. Excluding t from [2] of part 1 and [2] herein, we arrive at

$$\frac{d\bar{T}'_{p}}{d\bar{v}'_{p}} = \frac{Nu}{3Pr\bar{c}} \frac{1 - (1 + \bar{c}\gamma)\bar{T}'_{p}}{1 - (1 + \gamma)\bar{v}'_{p}},$$
[8]

where Nu and Pr are the Nusselt and Prandtl numbers, respectively.

For small particles one can take Nu = 2, i.e. the constant value. It we also neglect the variations in physical properties due to the temperature fluctuations, [8] can be integrated which yields

$$\bar{T}'_{p} = (1 + \gamma \bar{c})^{-1} \{ 1 - [1 - (1 + \gamma) \bar{v}'_{p}]^{\Omega} \},$$
[9]

where

$$\Omega = \frac{\mathrm{Nu}}{3\mathrm{Pr}\bar{c}} \frac{(1+\gamma\bar{c})}{(1+\gamma)}.$$

For a monodisperse system the temperature fluctuations are depicted in figure 2 as a function of the velocity fluctuations, with  $\Omega$  as a parameter. Clearly, for  $\Omega = 1$ , the temperature fluctuations are proportional to the velocity fluctuations. For  $\Omega > 1$  the temperature fluctuations are higher than the velocity fluctuations, and the inverse for  $\Omega < 1$ .

Now, by using [6], [9] and [4] from part 1, one obtains

$$\bar{T}^{1} = (1 + \gamma \bar{c})^{-1} \{ 1 + \gamma \bar{c} (\gamma^{-1} [\tilde{v}'(1 + \gamma) - 1])^{\Omega} \},$$
[10]

which relates the temperature fluctuations in the carrier fluid to the velocity fluctuations. Notice that [10], as well as [9], is completely universal (for a monodisperse system) since there was no hypothesis made on the structure of the turbulence.

From [2] and [6] we obtain

$$\frac{\mathrm{d}\bar{T}'_{\mathrm{p}}}{\mathrm{d}t} = \omega^* [1 - (1 + \gamma \bar{c})\bar{T}'_{\mathrm{p}}], \qquad [11]$$





Figure 1. Dependence of the temperature and velocity (curve  $\bar{c} = 1$ ) fluctuations of the particles on the total mass content of the admixture for various values of the ratio of the particles' and carrier fluid's specific heats.

Figure 2. Dependence of the particles' temperature fluctuations on the particles' velocity fluctuations for a monodisperse mixture.

where

$$\omega^* = 6 \frac{Nu\lambda}{c_{\rm p}d^2\rho_{\rm p}}$$

with  $\lambda$  being the thermal conductivity of the fluid.

Integrating [11] we obtain

$$\bar{T}'_{\rm p} = (1 + \gamma \bar{c})^{-1} (1 - e^{-k}), \qquad [12]$$

where  $k = \omega^* (1 + \gamma \bar{c})\tau^*$ , with  $\tau^*$  being the interaction time between the particles and the fluid, as in part 1. Using [9] one gets

$$k = 36 \frac{\Omega \beta}{\rho_{\rm pf} \text{Re}} \frac{(1+\gamma)^2}{1 - [1 - (1+\gamma \bar{c}) T_{\rm p}']^{1/\Omega}},$$
[13]

where  $\beta = l/d$ , Re =  $(|v'_0|d)/v$ ,  $\rho_{pf} = \rho_p/\rho$  and l is the mixing length.

Equations [12] and [13] gives the temperature fluctuations of the particles in terms of the hydrodynamic variables, physical properties  $(\Omega, \bar{c}, \rho_{pf})$  and loading  $(\gamma)$ .

## 4. BIDISPERSE MIXTURE

Consider a two-phase mixture—a carrier fluid in which particles of two sizes are suspended. The energy equations for the particles are

$$m_{\rm p}^{(1)}c_{\rm p}\frac{{\rm d}T_{\rm p}^{\prime(1)}}{{\rm d}t}=4\alpha_{\rm 1}f_{\rm 1}[T'-T_{\rm p}^{\prime(1)}]$$
[14a]

and

$$m_{\rm p}^{(2)}c_{\rm p}\frac{{\rm d}T_{\rm p}^{(2)}}{{\rm d}t}=4\alpha_2f_2[T'-T_{\rm p}^{\prime(2)}],\qquad [14b]$$

where the subscripts 1 and 2 correspond to coarse and fine particles, respectively.

By using [6] and [14] we find the absolute value of the temperature fluctuations of the coarse and fine particles:

$$\bar{T}_{p}^{\prime(1)} = (1 + \gamma \bar{c})^{-1} \left[ \left[ \left\{ (\varphi_{2} - \varphi_{1})^{-1} [\gamma_{1}^{-1} \bar{c}^{-1} (1 + \gamma \bar{c}) - \varphi_{2}] [\gamma_{1}^{-1} \bar{c}^{-1} (1 + \gamma_{2} \bar{c})] - \varphi_{1} \right\} \exp(-\gamma_{1} \bar{c} \varphi_{1} \bar{\tau}^{*}) \right. \\ \left. + \left[ \gamma_{1}^{-1} \bar{c}^{-1} (1 + \gamma \bar{c}) - \varphi_{1} \right] (\varphi_{2} - \varphi_{1})^{-1} [\gamma_{1}^{-1} \bar{c}^{-1} (1 + \gamma_{2} \bar{c}) - \varphi_{2}] \exp(-\gamma_{1} \bar{c}_{1} \varphi_{2} \bar{\tau}^{*}) + 1 \right]$$
[15a]

and

$$\overline{T}_{p}^{\prime(2)} = (1 + \gamma \bar{c})^{-1} \{ (\varphi_{2} - \varphi_{1})^{-1} [\gamma_{1}^{-1} \bar{c}^{-1} (1 + \gamma \bar{c}) - \varphi_{2}] \exp(-\gamma_{1} \bar{c} \varphi_{1} \bar{\tau}^{*}) - (\varphi_{1} - \varphi_{2})^{-1} [\gamma_{1}^{-1} \bar{c}^{-1} (1 + \gamma \bar{c}) - \varphi_{1}] \exp(-\gamma_{1} c \varphi_{2} \bar{\tau}^{*}) + 1 \},$$
[15b]

where

$$\bar{\tau}^* = \tau^* \omega_2^*,$$
  

$$\varphi_{1,2} = 0.5 \{ \gamma_1^{-1} \bar{c}^{-1} [(1 + \gamma_1 \bar{c})\epsilon + (1 + \gamma_2 \bar{c})] \pm \sqrt{\gamma_1^{-2} \bar{c}^{-2} [(1 + \gamma_1 \bar{c})\epsilon - (1 + \gamma_2 \bar{c})]^2 + 4\epsilon \gamma_1^{-1} \gamma_2} \}$$
and

 $\epsilon = d_2^2 d_1^2 \, .$ 

## 5. EFFECT OF RADIATION

Let us consider now the effect of radiation on the temperature fluctuations. The carrier fluid is assumed to be nonabsorbent and is laden by monodisperse particles.

The energy balance equations for the carrier fluid and the particles are

$$\frac{m_{\rm p}c}{\gamma}\frac{{\rm d}T'}{{\rm d}t} = -4\alpha f_{\rm p}(T'-T'_{\rm p}) \qquad [16a]$$





Figure 3. Temperature fluctuations of the carrier fluid and the particles as a function of the particles' fluctuation velocity ( $\tilde{c} = 0.13$ , Pr = 4).

Figure 4. Temperature fluctuations of the carrier fluid and the particles as a function of the particles' fluctuation velocity ( $\bar{c} = 2$ , Pr = 4).

and

$$m_{\rm p}c_{\rm p}\frac{{\rm d}T'_{\rm p}}{{\rm d}t} = 4lpha f_{\rm p}(T'-T'_{\rm p}) - q_{\rm R}\cdot 4f_{\rm p},$$
 [16b]

where  $q_{\rm R} = \sigma (T_{\rm p}^4 - T_{\rm w}^4)$ ,  $\sigma$  is the Stefan-Bolzmann constant and  $T_{\rm w}$  is the temperature of the wall surrounding the flow.

Consider two limiting cases: when the wall temperature  $T_w$  is much higher than that of the particles,  $T_w \gg T_p$ ; and when the temperature of the particles is much higher than the temperature of the walls,  $T_p \gg T_w$ .

We use [16a, b] to computer the dependence of the temperature fluctuations of the carrier fluid and the particles on various parameters for the first case when  $T_w \ge T_p$  and  $q_R = \text{const}$ :

$$\bar{T}' = 1 - \gamma \bar{c} (1 + \gamma \bar{c})^{-1} [1 - \bar{q}_{R} (1 + \gamma \bar{c})^{-1}] [1 - \exp(-N\tau^{*})] - \gamma \bar{c} (1 + \gamma c)^{-2} \bar{q}_{R} N\tau^{*}$$
[17a]

and

$$\overline{T}'_{p} = (1 + \gamma \bar{c})^{-1} \{ [1 - \bar{q}_{R} (1 + \gamma \bar{c})^{-1}] [1 - \exp(-N\tau^{*})] - \gamma \bar{c} (1 + \gamma \bar{c})^{-1} \bar{q}_{R} N\tau^{*} \},$$
[17b]

where

$$N = \omega^* (1 + \gamma \cdot \bar{c})$$
 and  $\bar{q}_{R} = -\frac{\sigma T_w^4}{\alpha T_0'}$ 

The second limiting case, when  $T_p \ge T_w$ , is described by the system of nonlinear equations [16a, b], which may be integrated numerically.

#### 6. RESULTS AND DISCUSSION

The results obtained indicate that the intensity of the temperature fluctuations in a two-phase mixture depends on a number of parameters: the physical properties of the fluid and particles, the total mass contents of the particles and the fraction of the particles of each size etc. However, the intensity of the temperature fluctuations mostly depends on the ratios of the particle to fluid densities and specific heats.

In figure 3, the temperature fluctuations are plotted as a function of the velocity fluctuations of the particles (monodisperse mixture with loading as a parameter). It is seen that an increase in the particles' velocity fluctuations is accompanied by an increase in the particles' temperature



Figure 5. The particles' temperature fluctuations for various values of Pr ( $\hat{c} = 1, \gamma = 1$ ).



Figure 6. The carrier fluid's temperature fluctuations as a function of the particles' velocity fluctuations for various values of Pr ( $\bar{c} = 1, \gamma = 1$ ).

fluctuations and a decrease in those of the carrier fluid. At some points the curves intersect and this point is the state of equilibrium of the system, i.e. temperature difference  $\Delta T' = 0$ . An increase in the total mass content  $\gamma$  leads to a decrease in the equilibrium temperature.

While figure 3 is computed for a specific heat ratio  $\bar{c} = 0.13$ , in figure 4 the ratio is  $\bar{c} = 2.0$ . A significant difference in the shape of the curves can be observed!

The effect of the Prandtl number on the dependence of the particles' (figure 5) and the fluid's (figure 6) temperature fluctuations on the particles velocity fluctuation is also seen to be substantial. For high values of the specific heat of the carrier fluid the particles' temperature fluctuations are very small and virtually do not vary. At small Prandtl number and low particle velocity fluctuations, a small increase in these causes a sharp increase in the particles' temperature fluctuations.

The dependence of the particles' temperature fluctuation on their velocity fluctuations for various values of the ratios of specific heats is depicted in figure 7. As the particles' specific heat increases, their temperature fluctuations decrease. At small values of  $\bar{c}$ , the temperature fluctuations are maximal.

In figure 8 the temperature fluctuations are plotted vs the Prandtl number. For very low Prandtl number (liquid metals) the fluctuations of the fluid and particles are equal and independent of the



Figure 7. The particles' temperature fluctuations as a function of the particles' velocity fluctuations for various values of the particles' and carrier fluid's specific heats  $(\gamma = 1, Pr = 4)$ .



Figure 8. Dependences of the carrier fluid's and the particles' temperature fluctuations on Pr  $(\beta = 75, \rho_{pf} = 2 \cdot 10^3, \text{Re} = 1, \gamma = 1, \text{Pr} = 4).$ 





Figure 9. The carrier fluid's and the particles' temperature fluctuations as functions of the particles' and the fluid's specific heats ratio ( $\beta = 75$ ,  $\rho_{pf} = 2 \cdot 10^3$ , Re = 1, Pr = 4).

Figure 10. The carrier fluid's and the particles' temperature fluctuations as functions of the total mass content of the admixture ( $\rho_{pf} = 2 \cdot 10^3$ , Re = 1,  $\bar{c} = 0.5$ , Pr = 4): (1)  $\beta = 50$ ; (2)  $\beta = 75$ ; (3)  $\beta = 100$ .

Prandtl number. For very large values of the Prandtl number (oils) the fluctuations of the temperature of the carrier fluid are very large and those of the particles are very small.

In figure 9 the temperature fluctuations are plotted as a function of the ratio of specific heats  $\bar{c}$ . An increase in the particles' specific heat leads to a decrease in the temperature fluctuations, and an increase in the thermal nonequilibrium. This effect was mentioned earlier by Shraiber *et al.* (1990).

The effect of the total particle mass content on the temperature fluctuations is shown in figure 10. The maxima in the curves for the particle temperature fluctuations  $\overline{T}'_{p}$  are due to the conflicting effect the loading has. On the one hand, increasing  $\gamma$  leads to a decrease in the particles' velocity fluctuations, and consequently to an increase in their interaction time, leading to an increase in the temperature fluctuations. On the other hand, increasing  $\gamma$  leads to an increase in the total mass capacity of the admixture, and consequently to a decrease in the temperature fluctuations, as is observed in figure 3.

The temperature fluctuations in a polydisperse mixture are shown in figure 11, where the





Figure 11. The carrier fluid's (T) and the coarse particles' ( $T_p^{(1)}$ ) temperature fluctuations as a function of the fine particles' temperature fluctuations  $T_p^{(2)}$  for various values of the total mass content in a polydisperse mixture  $[T = (1 + \gamma \bar{c})^{-1}].$ 

Figure 12. Dependence of the carrier fluid's temperature fluctuations on the particles' temperature fluctuations: (1)  $|\bar{q}_{R}| = 0.25$ , (2)  $|\bar{q}_{R}| = 0.5$ , (3)  $|\bar{q}_{R}| = 1$ , (4)  $|\bar{q}_{R}| = 2$  ( $T_{w} \ge T_{p}$ ); (1')  $|\bar{q}_{R}| = 0.125$ , (2')  $|\bar{q}_{R}| = 0.25$ , (3')  $|\bar{q}_{R}| = 0.5$  ( $T_{w} \ll T_{p}$ ).



Figure 13. Dependence of the carrier fluid's (2) and the particles' (1) temperature fluctuations on the parameter  $N\tau^*$  $(\gamma \bar{c} = 1, |\bar{q}|_R = 1; T_w \gg T_p).$ 



Figure 14. Dependence of the carrier fluid's and the particles' temperature fluctuations on the parameter  $|\bar{q}_R|$  $(T_w \ge T_p)$ : (1)  $\bar{T}_p$ , (1')  $\bar{T}'$   $(N\tau^* = 0.6)$ ; (2)  $\bar{T}_p$ , (2')  $\bar{T}'$  $(N\tau^* = 1.8)$ ; (3)  $\bar{T}_p$ , (3')  $\bar{T}'$   $(N\tau^* = 3.0)$ .

temperature fluctuations of the carrier fluid are plotted as a function of the temperature fluctuations of the fine particles, with  $\gamma \bar{c}$  as a parameter.

The effect of radiation is demonstrated in figures 12-14 with  $T_p \gg T_w$  and  $T_w \gg T_p$  for a monodisperse admixture. It is seen that radiation strongly affects the temperature fluctuations of the particles and the carrier fluid.

In figure 12 the temperature fluctuations of the carrier fluid are plotted as a function of the particle temperature fluctuations for various values of the parameter  $\bar{q}_R(\bar{q}_R \ge 0)$ . The graph shows a line corresponding to the flow without radiation, when the fluctuation part of the enthalpy  $J'_m = \bar{T}' + \gamma \bar{c} \bar{T}'_p$  of the fluid element is constant during its lifetime. The point M on this line, corresponding to the limit regime  $\bar{T}' = \bar{T}'_p$ , restricts the domain of the physically realizable states of the system without radiation (the section MN).

The line  $\bar{q}_{R} = 0$  divides the parametrical plane  $\bar{T}'\bar{T}'_{p}$  on the two regions corresponding to the regimes with  $T_{w} > T_{p}$  (A and B) and with  $T_{w} < T_{p}$  (C and D). The curve  $\bar{T}'(\bar{T}'_{p})$  for  $\bar{q}_{R} \leq 0$  come from the point N, corresponding to the initial state of the system  $(t = 0, \bar{T}' = 1, \bar{T}'_{p} = 0)$ . Each point on these curves has its own value of  $\tau^*$ , and this value increases the farther the point is from point N.

In figure 12 the lines passing via the curves'  $\overline{T}'(\overline{T}'_p)$  extrema are also shown. These lines correspond to the conditions  $d\overline{T}'/d\overline{T}'_p = 0$  ( $\overline{q}_R < 0$ ) and  $d\overline{T}'_p/d\overline{T}' = 0$  ( $\overline{q}_R > 0$ ). In the domain A, bounded by the curves NM and Mm (small  $\tau^*$ ), the carrier fluid temperature fluctuations are larger than those of the particles. In this case the particles' temperature fluctuations increase both due to the convective heat transfer and radiation. The carrier fluid temperature fluctuations in this domain decrease due to the convective heat transfer. On the line Mm, the carrier fluid and particles' temperature fluctuations are equal. In the domain B, the direction of the convective heat transfer changes, i.e. the particles' temperature fluctuations are larger than those of the carrier fluid. The latter leads to an increase in the fluid's temperature fluctuations.

In the domain  $C(\bar{q}_R > 0)$ , the convective heat removal from the carrier fluid to the particles results in a decrease in the carrier fluid temperature fluctuations. This phenomenon was observed when investigating the effect of radiative heat transfer on temperature fluctuations (Lisin 1988). For larger  $\tau^*$ , when the heat losses increase, the carrier fluid and particles' temperature fluctuations decrease (domain D). Due to the latter, the carrier fluid's temperature fluctuations are larger than those of the particles.

In figure 13, the carrier fluid and the particles' temperature fluctuations are plotted as functions of the parameter  $N\tau^*$ . The particles' temperature fluctuations gradually increase with an increase in the interaction time  $\tau^*$ . As to the dependence of  $\overline{T'}(N\tau^*)$ , it has an extremum at the point where the convective heat flux changes its direction.

The effect of the radiative heat flux on the carrier fluid's and the particles' temperature fluctuations is demonstrated in figure 14. It is shown that the radiative flux increase (in the case  $T_w > T_p$ ) leads an increase in the carrier fluid's and the particles' temperature fluctuations.

## 7. CONCLUSIONS

The intensity of the temperature fluctuations in two-phase flows of monodisperse and polydisperse mixtures have been studied using the modified mixing-length theory. For various types of two-phase flows the peculiarities of the temperature fluctuations fields have been calculated and the dependence of the results on the governing parameters studied. It is shown that the level of the temperature fluctuations is strongly affected by the total mass content of the admixture, its composition and specific heat and the particle sizes, as well as the viscosity and thermal conductivity of the carrier fluid.

The following results have been obtained:

- 1. The intensity of temperature fluctuations in two-phase flows (carrier fluid and the particles of one or two fractions) is determined by the following parameters: the total mass content of the admixture, the mass contents of the particles of the fine and coarse fractions, their diameter ratio, the particles' and the carrier fluid's specific heats and densities ratios, the particles Reynolds, Nusselt and Prandtl numbers and the ratio of the mixing length to the diameter of one group.
- 2. An increase in the total mass content of the admixture and its specific heat leads to a reduction in the intensity of the temperature fluctuations of the carrier fluid.
- 3. An increase in the Prandtl number leads to an increase in the carrier fluid's temperature fluctuations and a decrease in the particles' temperature fluctuations.
- 4. Radiation strongly affects the temperature fluctuations of the particles and the carrier fluid.

In conclusion, we emphasize that the modified mixing-length theory used in part 1 and in the present study makes it possible to describe a number of very important characteristics of two-phase turbulent flows, which manifest the effects of the viscous, thermal and inertial forces.

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